Learning in the Presence of Strategy

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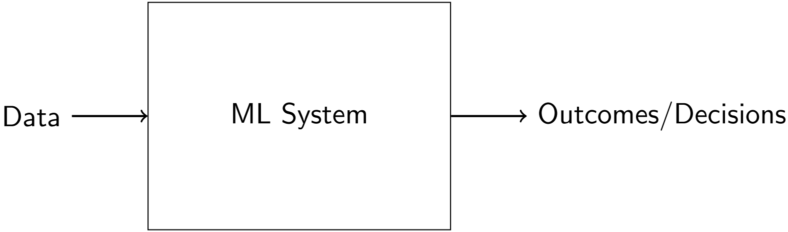
Classical ML



Classical ML

Historical Loans →

→ Loan decision

Classical ML

Medical History →  → Health Risk

Learning in the Presence of Strategy

- Agents with different goals, information and strategies interact with AI/ML algorithms

Learning in the Presence of Strategy

- Agents with different goals, information and strategies interact with AI/ML algorithms
- Agents are rational; take action that is best given their information
- Misalignment in objectives (also, sometimes information) leads to inefficiency

Strategic Learning

Classical vs Strategic Learning

- Let f be a binary classifier

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  - If $y' = y$ then it is called gaming [a]
  - If also $y' \neq y$; the rule f is called performative prediction

[a] Strategic Classification, Hardt, Maggiddo, Papadimitriou, Wooters. ITCS 2016

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- Agents/Users want favourable outcome; 1 if classified positively and 0 otherwise.

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- System's payoff: $\mathbb{E}_{(x,y)\sim\mathcal{D}}[f(\Delta_f(x)) = y]$.
  Throughout this talk we will consider strategic error.

$$f_s^* \in \arg\min_{f \in \mathcal{F}} \mathbb{E}_{(x,y)\sim\mathcal{D}}[f(\Delta_f(x)) \neq y]$$

Systems goal: Find f^* that adjusts to distribution shift in test data.

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1. Health risk predictions
2. Bank loan approvals
3. Corporate hiring/promotions ...

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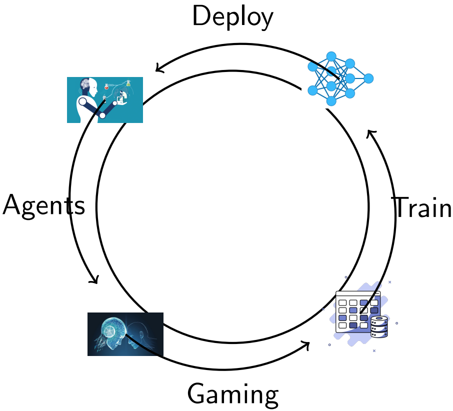

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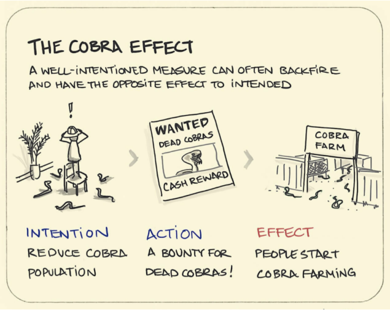
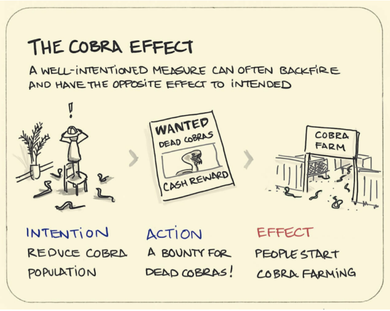


THE COBRA EFFECT
A WELL-INTENTIONED MEASURE CAN OFTEN BACKFIRE AND HAVE THE OPPOSITE EFFECT TO INTENDED
WANTED
DEAD COBRAS
CASH REWARD
COBRA FARM
INTENTION
REDUCE COBRA POPULATION
ACTION
A BOUNTY FOR DEAD COBRAS!
EFFECT
PEOPLE START COBRA FARMING

Gaming



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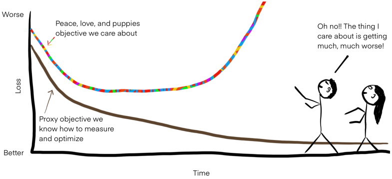

Worse
Peace, love, and puppies objective we care about
Oh no!! The thing I care about is getting much, much worse!
Loss
Proxy objective we know how to measure and optimize
Better
Time

Warmup

- Classical vs strategic classification: $\mathcal{D}_{train} \neq \mathcal{D}_{test}$ in strategic classification

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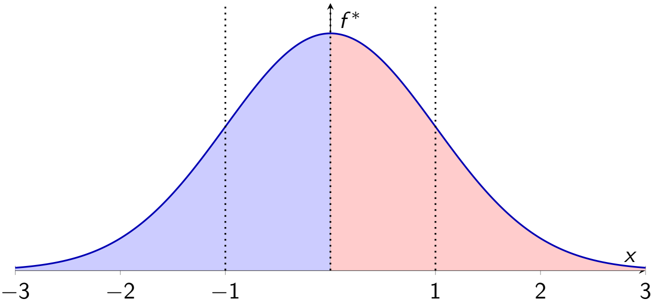
- $\mathcal{D}_{test}$ is a obtained from implemented classifier f and $\mathcal{D}_{train}$



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- Game theoretic interpretation: Two players, System and User(s) play following Stackelberg game

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  - System learns a classifier f from training data $\mathcal{D}_{train}$
  - System declares f publicly
  - User, on observing f, misreport (at cost) her features to obtain the desired outcome from f

Goal: To minimize risk under strategic data distribution shift (strategic error).

Learning Objective

Objective Function

$$\min_{f \in \mathcal{F}} \frac{1}{m} \sum_{i=1}^{n} \mathbb{1}[f(\Delta_f(x_i)) \neq y_i]$$

$$\textit{with} \quad \Delta_f(x) = \arg\max_{x'} f(x') - c(x, x')$$

- cost function important ($\Delta_f(x)$ unique?)
- Strategic Agents: Move to the point y on the boundary of the classifier at cost $c(x, y)$ only if $c(x, y) \leq 2$.
- Nested min-max problem

Examples

- $c(x,y) = ||y - x||_2$, $\mathcal{F}$ linear classifiers; $f(x) = w^T x$. Then

$$\Delta_w(x) = \begin{cases} x & \text{if } w^T x \geq 0 \text{ or } c(x;w) > 2. \\ proj(x;w) & \text{otherwise} \end{cases} \quad (1)$$

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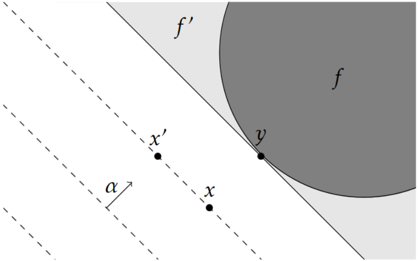
- (Separable) $c(x,y) = \max(0, c_2(y) - c_1(x))$ with $c_2(X) \subseteq c_2(X)$ e.g., $\langle \alpha, y - x \rangle_+$.



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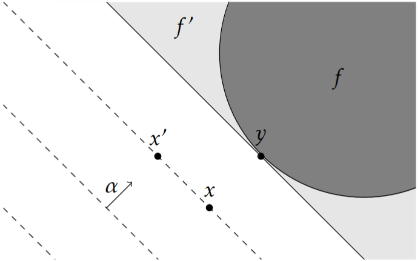

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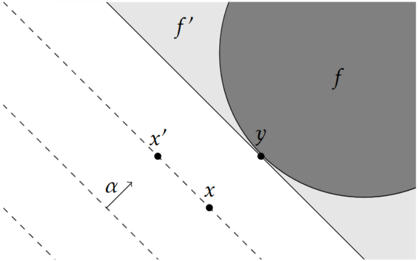


- (Decomposable) $c(x,y) = \sum_{i=1}^{d} \alpha_i |g_i(y_i) - g_i(x_i)|_+$ more on it later

Learnability Issues [1]

- Induced class $\mathcal{F}_\Delta = \{f(\Delta_f(x)) : f \in \mathcal{F}\}$.
- $SVC(\mathcal{F}) := VC(\mathcal{F}_\Delta)$.
- Cost is important
  - instance-invariant cost (cost of altering x is the same for all x); $SVC(\mathcal{F}) \approx VC(\mathcal{F})$ for linear f
  - instance-wise cost: SVC is ∞ even for linear classifiers.

[1] Sundaram et al. PAC-learning for Strategic Classification, JMLR 2023

Learning Results: Strategic ERM

Definition (Strategy Robust Learning)

An algorithm $\mathcal{A}$ is a strategy-robust learning algorithm for a class of cost functions $\mathcal{C}$ if for all distributions $\mathcal{D}$, for all class probabiliy functions η, all $c \in \mathcal{C}$ and for all ε and δ, given a description of c and access to labeled examples of the form $(x, \eta(x))$, where $x \sim \mathcal{D}$, $\mathcal{A}$ produces a classifier $f : \mathcal{X} \to \{-1, 1\}$ so that, with probability at least $1 - \delta$ over the samples,

$$Pr_{x \sim \mathcal{D}}(\eta(x) = f(\Delta_f(x))) > OPT(\mathcal{D}, c) - \varepsilon$$

Theorem

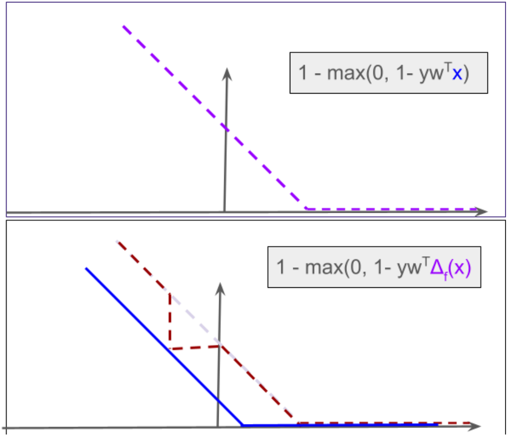
Let $\mathcal{H}$ be a concept class, $\mathcal{D}$ be a distribution and c be a separable cost function. Further, let m denote the number of samples and suppose

$$\mathcal{R}_m(\mathcal{H}) + 2\sqrt{\frac{\log(m+1)}{m}} + \sqrt{\frac{\log(2/\delta)}{8m}} \leq \frac{\varepsilon}{8}. \tag{2}$$

Then SERM outputs f such that w.p. atleast $1-\delta$,
$\mathbb{P}_{x \in \mathcal{D}}(\eta(x) = f(\Delta(x))) \geq \textsc{Opt}(\mathcal{D}, c) - \varepsilon.$

More on optimization Problem

- Why cant we use surrogate loss functions (such as hinge loss, log loss)



Variation 1: SC in the Dark [2]

[2] Ghalme et al. Strategic Classification in the Dark, ICML 2021.

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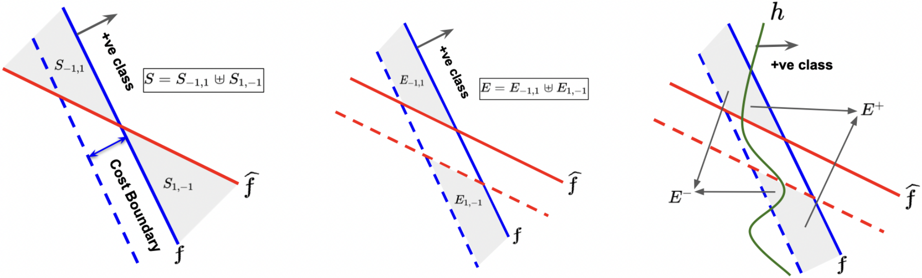
Who is in the dark? By making f public, System can anticipate agents' response better (and construct robust f). By keeping f private, System is also in the dark as partially informed users may lead to unpredictable response.

Price of Opacity



Main Results

Definition (Price of Opacity (POP))

$$POP(f, f') := \textsc{Err}(f, f') - \textsc{Err}(f, f).$$

Here f is the System's classifier and f' is the classifier Agents' classifier (Agent responds to f').

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Theorem (POP characterization)

If $\mathbb{P}_{x \sim \mathcal{D}}(x \in E) > 2\textsc{Err}(f^*, f^*) + 2\varepsilon$*, then* $POP > 0$.

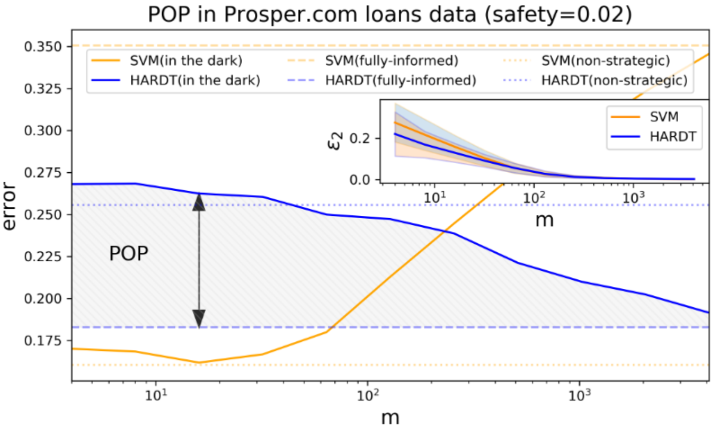

Figure: Price of Opacity is positive and decreases with the training samples m used to construct $\widehat{f}$.

Performative Prediction [3]

[3] Perdomo et al. Performative Prediction. ICML 2020

Variation 2: Performative Prediction

- SC assumption: Labels are immutable
- Performative Prediction: The (joint) distribution $\mathcal{D}$ changes to $D(\theta)$ where θ represents the parameters of the classification rule.
- Predictive Optimal policy $PO = \arg\min_{f \in \mathcal{F}} \mathbb{E}_{Z \sim \mathcal{D}(\theta))} \ell(Z; \theta)$

Performative Prediction

- The deployed classifier has performative effect on qualification (improves with the cost/effort spent on obtaining positive outcome).
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Definition (Performative Stability)

A model $f_{\theta_{ps}}$ is called performatively stable if

$$\theta_{PS} = \arg\min_{\theta} \mathbb{E}_{Z\sim\mathcal{D}(\theta_{PS})}\ell(z;\theta)) \tag{4}$$

Results: Performative Predictions

Theorem (Informal)

If the loss is smooth, strongly convex, and the mapping $\mathcal{D}(.)$ is sufficiently Lipschitz, then repeated risk minimization converges to performative stability at a linear rate.

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Theorem (Informal)

If the loss is Lipschitz and strongly convex, and the map $\mathcal{D}()$ is Lipschitz, all stable points and performative optima lie in a small neighbourhood around each other.

Improvement Aware Strategic Classification

Improvement Aware SC

- Assume that η is component-wise increasing in x [4] and the cost function is decomposable.

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  - $\mathrm{ERR}_{\mathtt{imp}}(f) := \mathbb{E}_{(x,y)\sim\mathcal{D}}[\mathbb{1}(f(\Delta_f(x)) \neq y')]$ where $y' \sim \mathit{Bernoulli}(\eta(\Delta_f(x)))$
- Goal: Find optimal Strategy aware classifier i.e., $f^*_{\mathtt{imp}} \in \arg\min_{f\in\mathcal{F}} \mathrm{ERR}_{\mathtt{imp}}(f)$
- Also assume that the Bayes optimal classifier is also Linear.

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Results

Theorem

$f^*(x; w, b) = f_s^*(x; w, b + \phi)$ *with* $\phi = \max_i \frac{w_i}{\alpha_i}$.

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- f^* represents an optimal (linear) classifier with pristine, non-manipulated data; a naive approach
- Optimal classifier with manipulated data with no-improvement; a pessimistic approach

Theorem

$f^*_{imp(x)} = f^*(x; w, b + \phi')$ *where* $\phi' \in [0, \max_i \frac{w_i}{\alpha_i}]$. *Furthermore,* $err_{imp}(f_s^*) \leq err_{imp}(f^*)$.

- The improvement aware classifier lies between naive and pessimistic classifiers.
- Also, the pessimistic classifier is a more reliable proxy for improvement aware classifier over a naive classfier.

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Thank you!

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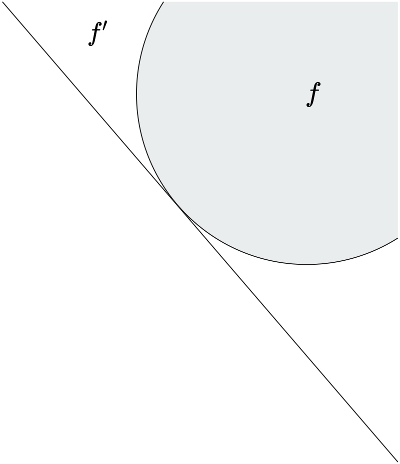


Algorithm 1: $\mathcal{A}$: gaming-robust classification algorithm for separable cost functions

1. **Inputs:** Labeled examples $(x_1, h(x_1)), \ldots, (x_m, h(x_m))$ from $x_i \sim \mathcal{D}$ i.i.d.. Also, a description of a separable cost function $c(x,y) = \max\{0, c_2(y) - c_1(x)\}$. ⇦
2. For $i = 1, \ldots, m$, let

$$t_i := c_1(x_i)$$

$$s_i := \begin{cases} \max\left(c_2(X) \cap [t_i, t_i + 2]\right) & c_2(X) \cap [t_i, t_i + 2] \neq \emptyset \\ \infty & c_2(X) \cap [t_i, t_i + 2] = \emptyset \,. \end{cases}$$

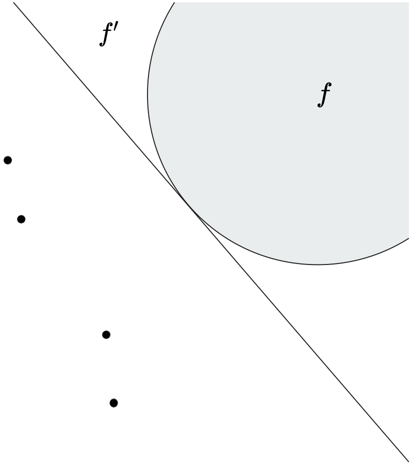
   For convenience, set $s_{m+1} = \infty$.
3. Compute

$$\widehat{\mathrm{err}}(s_i) := \frac{1}{m} \sum_{j=1}^{m} \mathbf{1}\left\{h(x_j) \neq c_1[s_i - 2](x_j)\right\}.$$

4. Find i^*, $1 \leqslant i^* \leqslant m+1$, that minimizes $\widehat{\mathrm{err}}(s_i)$.
5. **Return:** $f := c_2[s_{i^*}]$.

1

Strategic ERM



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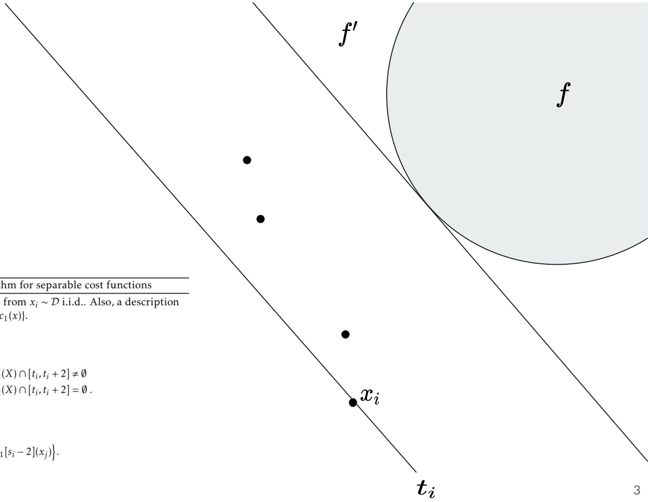
2

Strategic ERM



Algorithm 1: $\mathcal{A}$: gaming-robust classification algorithm for separable cost functions

1 **Inputs:** Labeled examples $(x_1, h(x_1)), \ldots, (x_m, h(x_m))$ from $x_i \sim \mathcal{D}$ i.i.d.. Also, a description of a separable cost function $c(x,y) = \max\{0, c_2(y) - c_1(x)\}$.

2 For $i = 1, \ldots, m$, let

$$t_i := c_1(x_i) \ \Leftarrow$$

$$s_i := \begin{cases} \max(c_2(X) \cap [t_i, t_i + 2]) & c_2(X) \cap [t_i, t_i + 2] \neq \emptyset \\ \infty & c_2(X) \cap [t_i, t_i + 2] = \emptyset \,. \end{cases}$$

For convenience, set $s_{m+1} = \infty$.

3 Compute

$$\widehat{\text{err}}(s_i) := \frac{1}{m} \sum_{j=1}^{m} \mathbf{1}\left\{h(x_j) \neq c_1[s_i - 2](x_j)\right\}.$$

4 Find i^*, $1 \leqslant i^* \leqslant m+1$, that minimizes $\widehat{\text{err}}(s_i)$.

5 **Return:** $f := c_2[s_{i^*}]$.

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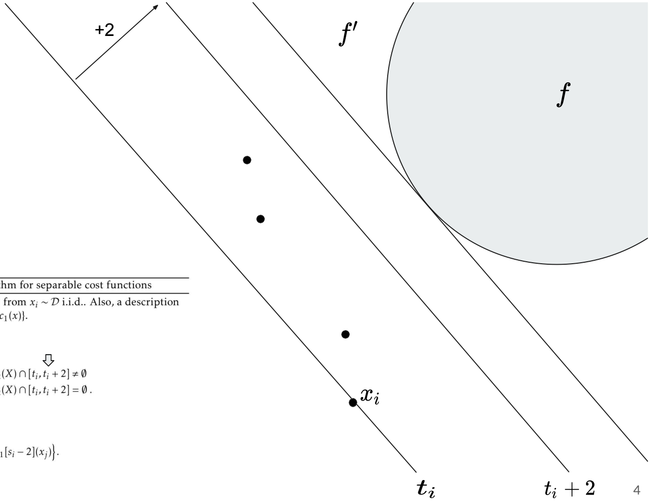

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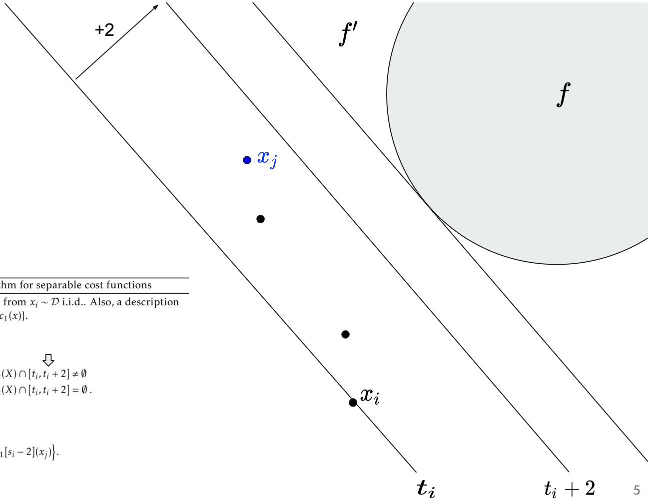

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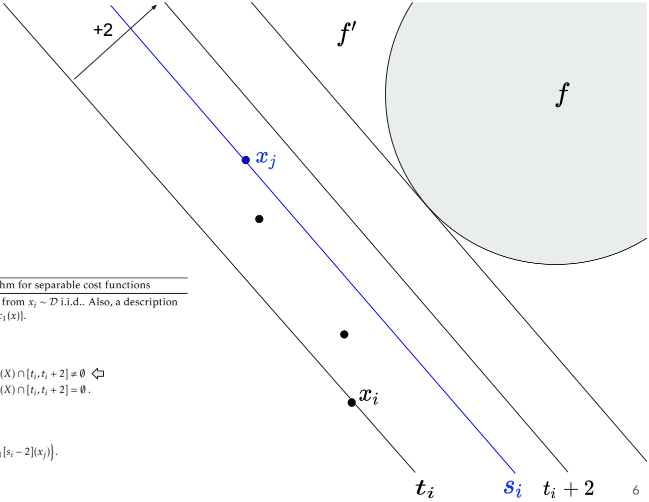

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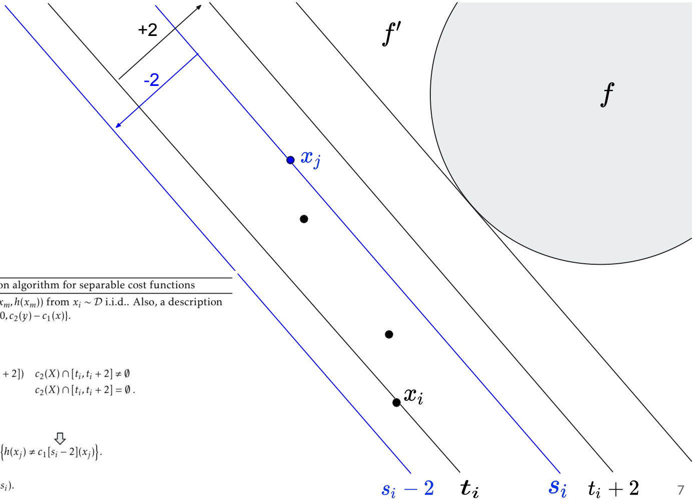

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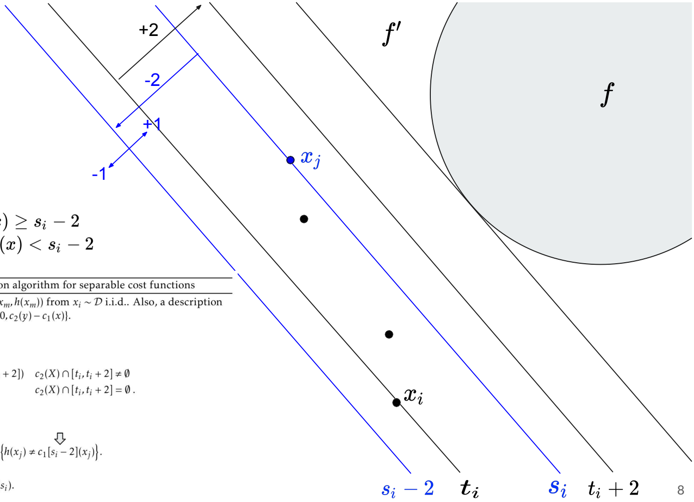

$$c_1[s_i - 2](x) = \begin{cases} 1, \text{ if } c_1(x) \geq s_i - 2 \\ -1, \text{ if } c_1(x) < s_i - 2 \end{cases}$$

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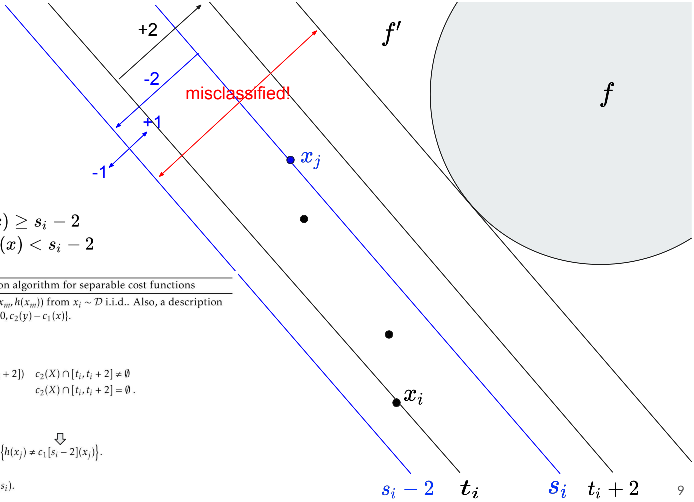

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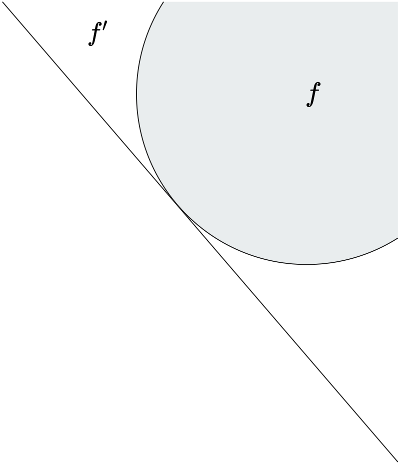

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$$c_2[s_i^*](x) = \begin{cases} 1 \text{ if } c_2(x) \geq s_i^* \\ -1 \text{ if } c_2(x) < s_i^* \end{cases}$$

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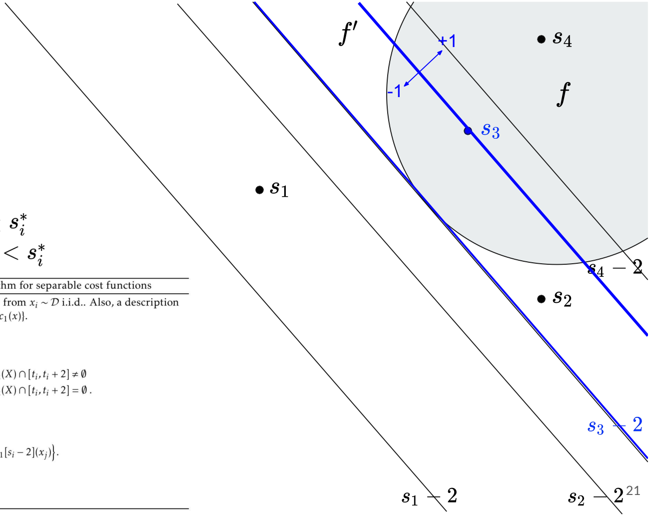
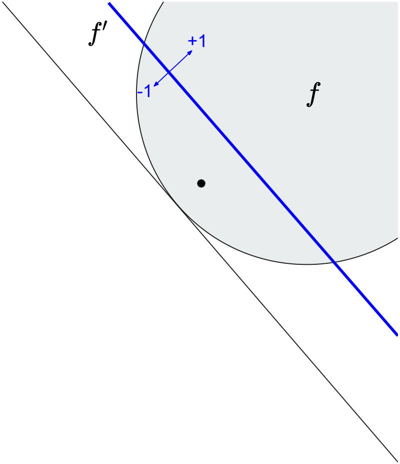


New input 1



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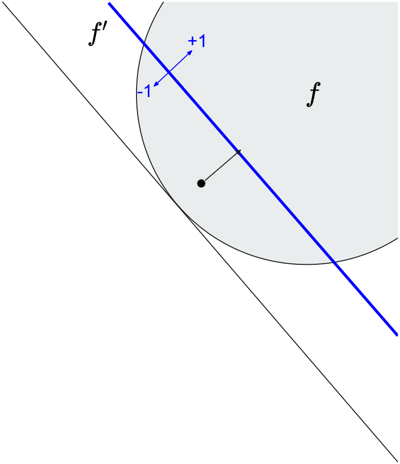

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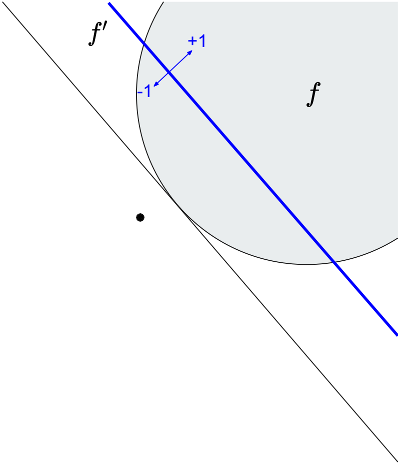
New input 2



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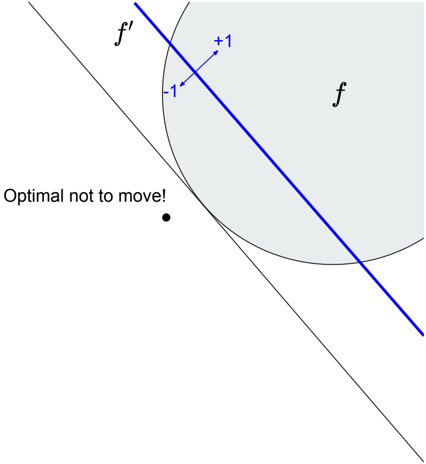
New input 2



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